PRICING OF WEATHER DERIVATIVES: HOW TO CALIBRATE EXPECTED LOSSES AND SUITABLE MARGINS

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Agenda

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1. **Introduction**
2. Modelling Weather Time Series – Comparison of four Models
   - Burn Analysis
   - Two Sinus Models
   - Spline Model
3. How to Calculate Risk Margins
   - Classical Methods for Arbitrage-Free and Complete Markets
   - Extensions for Inclomplete Markets
   - Non-Hedgeable Risks

Introduction
Temperature derivatives and reinsurance

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Why might a reinsurer be interested in Heating and Cooling Degree Days contracts?

- Additional investment class
  However: may be correlated to main business
- Provide service to cedents
  Primary insurance companies are often not allowed to invest in HDD or CDD
  Munich Re can structure a reinsurance contract according to these contracts
- Mitigate insurance risks
  Hurricanes and hail storms may be correlated to CDD
  Freezing rain may be correlated to HDD

Innovative investment and new products based on weather derivatives

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Comparison of three models
General approaches

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To derive an estimate (or future expectation) for the payoff of a HDD or CDD contract, three typical models types of models are common:

- Burn Analysis:
  Use historical payoff time series to estimate future expected value
- Index Modelling:
  Use historical index data for the HDD or CDD times series, estimate the expected future index and calculate the expected payoff
- Daily Simulation:
  Fit a stochastic model to the daily temperature time series and estimate the index and the payoff by simulating the future temperature.

Complexity increases, but does also the quality of the results?

Comparison of three models
Burn analysis and index modelling

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For the times series Y_1, Y_2, …, Y_n of payoff or index values in the last n years use a linear model to predict the future

$$Y_i = \beta_0 + \beta_1 \cdot i + \varepsilon_i, \quad i = 1, \ldots, n$$

Parameters can be directly estimated and assuming that ε_i are iid normal, we can predict and simulate the next value Y_{n+1}

We prefer modelling index values as payoffs are often truncated by 0 or the maximal payoff.

Easily implemented with standard tools

Comparison of three models
Sinus models

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The Alaton model and the Benth model both assume that the temperature T_t follows a Hull-White type stochastic process over time

$$\mathrm{d}T_t = \left(a(\theta_t - T_t) + \frac{\mathrm{d}\theta_t}{\mathrm{d}t} \right) \mathrm{d}t + \sigma_t \mathrm{d}W_t, \quad t \geq 0$$

With mean reverting term θ_t, modelling the linear and seasonal trends:

Alaton: $\theta_t = A + Bt + C\sin(\omega t + \varphi) + Dt\sin(\omega t + \phi)$

Benth: $$\theta_t = a + bt + \sum_{i=1}^{I_1} a_i \sin(2i\pi(t - f_i)/365) + \sum_{j=1}^{J_1} b_j \cos(2j\pi(t - g_j)/365)$$

and variance σ_t^2:

Alaton: Variance is assumed to be constant

Benth: $$\sigma_t^2 = c + \sum_{i=1}^{I_2} c_i \sin(2i\pi t/365) + \sum_{j=1}^{J_2} d_j \cos(2j\pi t/365)$$

Comparison of three models
Sinus models

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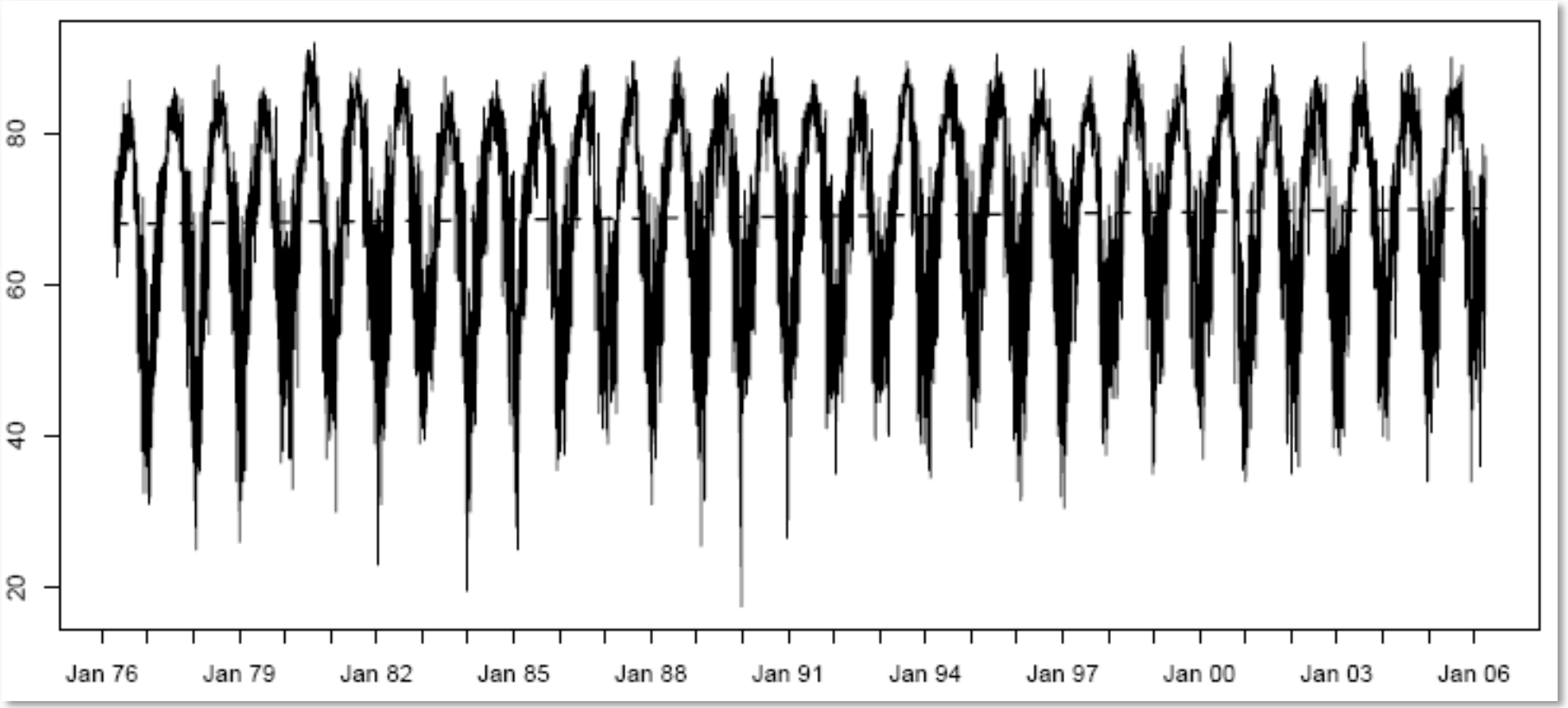
Temperature [°F] for Huston, Texas



Is sinus approximating weather time series well enough?

Comparison of three models
Spline model

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We generalize the sinus models by assuming that the temperature T_t follows a generic process

$$T_t = \mu_t + \sigma_t R_t$$

With μ_t and σ_t modelled using splines and the remaining residuals R_t are modelled with an autoregressive process.

Motivation:

- Splines are more flexible to fit to specific shapes observed in historic data
- We are able to jointly model the daily trends over the years on a surface days x years
- Not restricting ourselves on a stochastic process we have the full flexibility to choose any suitable autoregressive times series model that might fit best

Best fit with low number of parameters

Comparison of three models
Spline model: trend and seasonality

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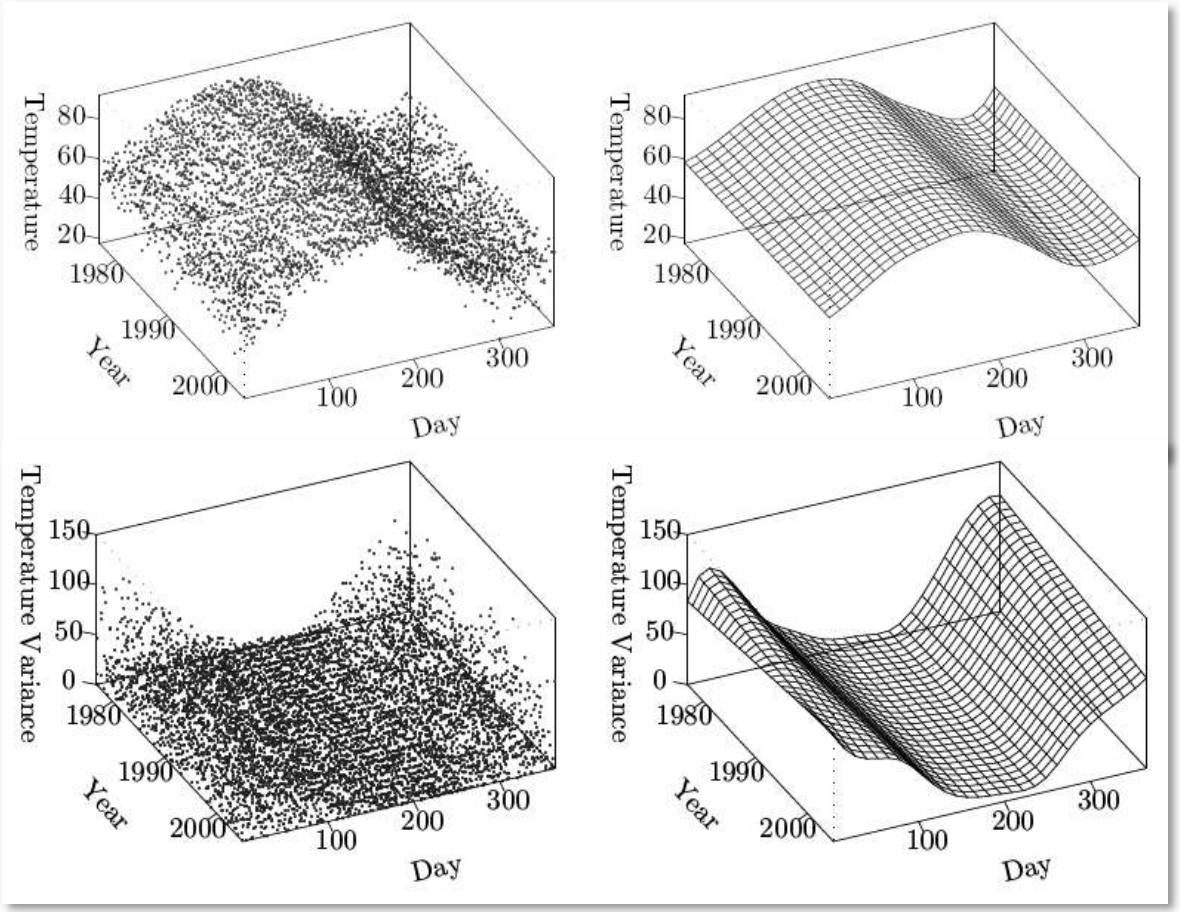
Temperature [°F] and approximation for Huston, Texas



- Linear trend for years
- Cubic spline with knots every 60 days for days
- Trend (above panel) and volatility (below panel) are well fitted
- Especially volatility shows different trends in different seasons and shape is not well approximated by sinus

Comparison of three models
Spline model: Residues

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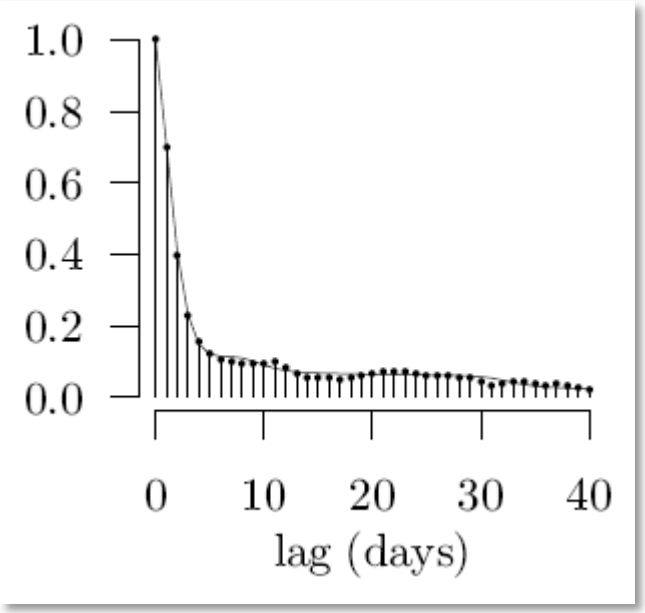
Autocorrelation of residues for Huston, Texas



- Autocorrelations of residues fall below 0.2 after four days, however, stay strictly positive for the next 40 days ▸ weather proverbs!
- AR(1) process decreases exponentially
- AROMA process provides long range autocorrelation by using moving averages
- Using max. four terms and fixing $m_1 = 1$ and $m_2 = 2$ we found iteratively $m_3 = 8$ and $m_4 = 31$

Autoregressive on Moving Averages (AROMA) process:

$$T_t = \phi_1 \bar{T}_{m_1,t} + \phi_2 \bar{T}_{m_2,t} + \ldots + \phi_r \bar{T}_{m_r,t} + W_t$$

$$\bar{T}_{m,t} = \frac{1}{m} \sum_{i=1}^{m} T_{t-i}$$

Comparison of three models

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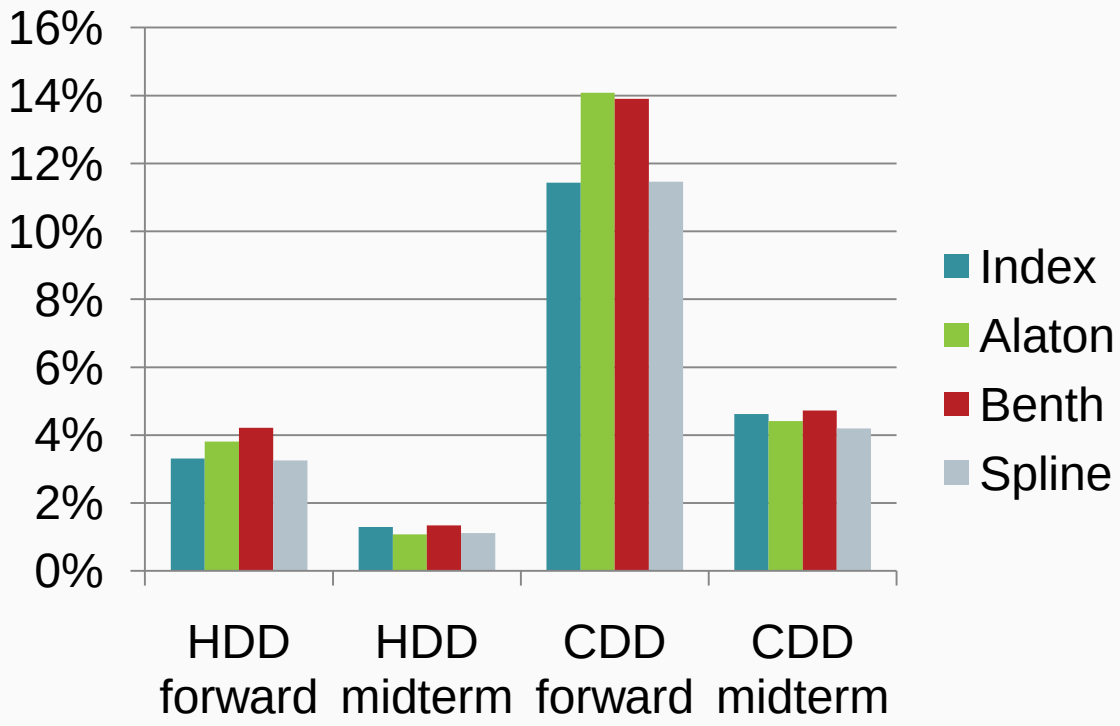
Model comparison on backtesting US weather stations



- Despite its simple form, the index model is always one of the best
- Benth model cannot even compete in the midterm valuation
- Alaton and Spline models provide best fit in midterm valuation
- Spline model also provides best fit in 180 days forward pricing

Spline model achieves best results for all purposes

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How to Calculate Risk Margins
Models from the Financial Industry

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- Black Scholes Pricing model:
  Weather index data is not tradable, hence hedge is impossible to construct
  ► This model will result in meaningless prices
- Models used in incomplete markets (e.g. super hedging, quadratic hedging):
  Again, the main problem is, that the index itself is not traded, hence only super hedging is applicable
  ► These models result in extreme prices
- Supply and demand approach:
  Currently, the market is not deep enough. Other methods will have to be used.
- Models used for non-hedgeable risks:
  These models are also typically used in the insurance industry (see next slide).

Classical financial market pricing methods are not directly applicable

How to Calculate Risk Margins
Models for non-hedgeable Risks

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Standard method for non-hedgeable risks is to

1. calculate the risk capital
   our models allow for calculating the VaR and ES of the payoff distribution

2. add the cost of capital as a margin
   typically 6% of the risk capital is used
   factor may depend on current financial situation, reputation and the need to raise capital

“Hedges” for the weather derivatives may be included in the capital model by providing diversification with other segments.

This way steering business and risk management will be as transparent as possible.

This approach is rather complex but transparently quantifies the real costs

THANK YOU FOR YOUR ATTENTION!

Dr. Frank Schiller, Head of CoC Direct Insurance

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