MODELING LONGEVITY RISK IN PRACTICE

Summer School on Risk Measurement and Control

July 2010, Rome

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Agenda

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- Stochastic mortality models
- Model comparison
- Impact on annuity products – comparison to Solvency II
- Internal models in the Solvency II setting

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STOCHASTIC MORTALITY MODELS

Widely used Models:
Mostly not Stochastic Models

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Increasing Complexity

Mortality	Longevity	CI	TPD and DI	LTC
No model at all	Always trends are modelled	No model at all	No model at all	Depending on guarantees
5-years regression	15-years and longer regression	5-years regression	5-years regression	5- to 15-years regression
Lee Carter	Lee Carter	Never seen	Never seen	Lee Carter
Typically no more complex models	Huge variety	Typically no more complex models	Typically no more complex models	Typically no more complex models

Models

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- Lee Carter model: $log(q_{x,t}) = \mathrm{a_x} + \mathrm{b_x}\kappa_\mathrm{t} + \mathrm{error}$

- Bayesian Lee Carter: Lee Carter including parameter uncertainty

- Cairns, Blake & Dowd (CBD) model:

$$logit(q_{x,t}) = \log\left(\frac{q_{x,t}}{1 - q_{x,t}}\right) = \kappa_\mathrm{t}^{(1)} + \kappa_\mathrm{t}^{(2)}(\mathrm{x} - \bar{\mathrm{x}}) + \mathrm{error}$$

► model κ_t , $\kappa_\mathrm{t}^{(1)}$, $\kappa_\mathrm{t}^{(2)}$ as time series processes, e.g. (bivariate) random walk with drift

► forecast κ_t , $\kappa_\mathrm{t}^{(1)}$, $\kappa_\mathrm{t}^{(2)}$ into the future

MODEL COMPARISON

Modeling mortality data from Italy – prediction quality

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Mortality.org: Italian mortality data for years 1960 to 2006 and ages 0 to 98

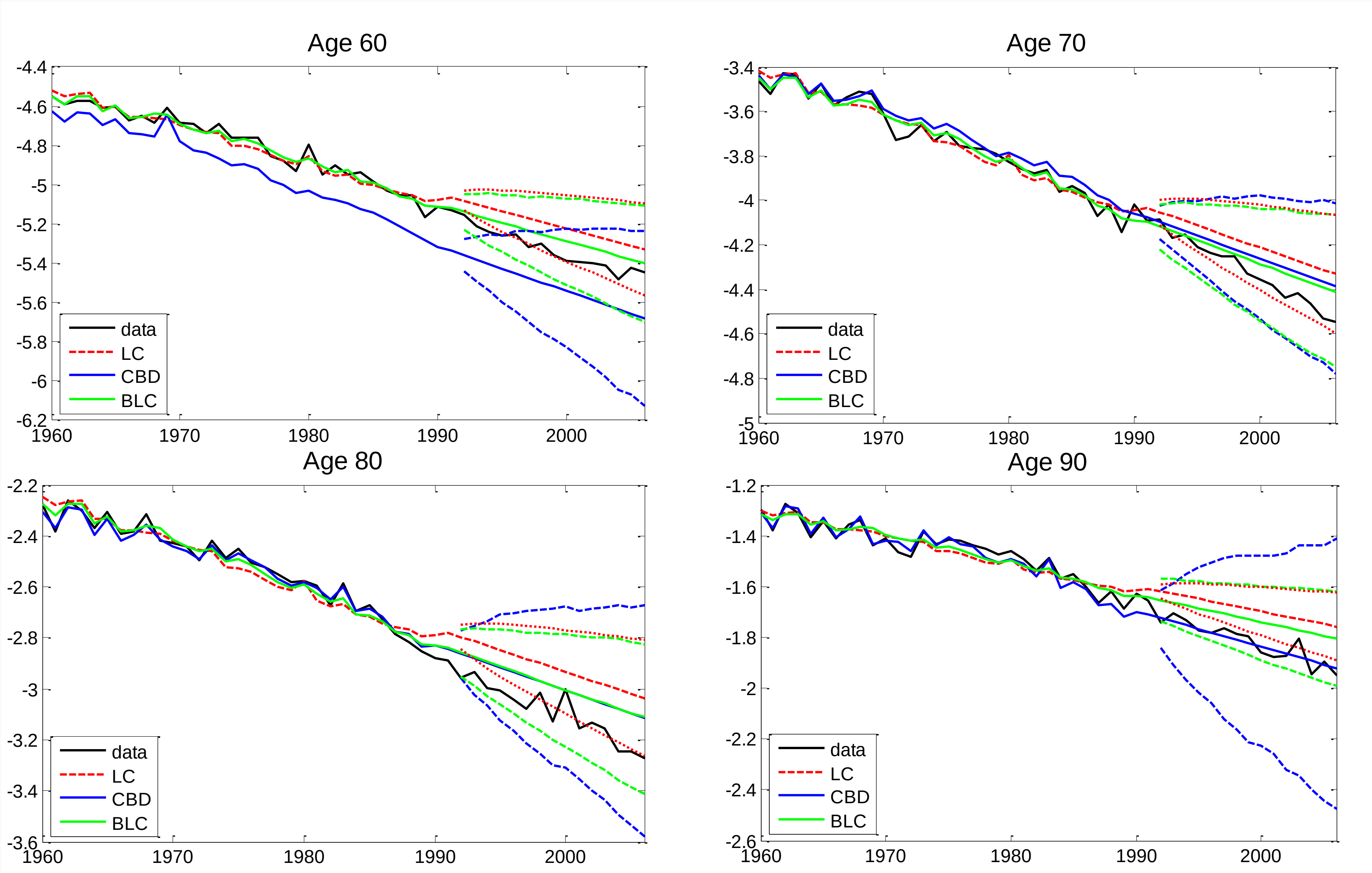
Comparing prediction quality of the models (backtesting):

- Fit models to data for years 1960 to 1991
- Forecast mortality for years 1992 to 2006

Test implicit assumptions made in the models:

- LC assumes time-independent age effects (bilinear model) and thus that mortality improvements at all ages are perfectly correlated
- CBD model assumes that logit of rates is linear
fitted only to relevant ages 60 to 98

Females



Age 60
-4.4
-4.6
-4.8
-5
-5.2
-5.4
-5.6
-5.8
-6
-6.2
1960
1970
1980
1990
2000
data
LC
CBD
BLC
Age 70
-3.4
-3.6
-3.8
-4
-4.2
-4.4
-4.6
-4.8
-5
1960
1970
1980
1990
2000
data
LC
CBD
BLC
Age 80
-2.2
-2.4
-2.6
-2.8
-3
-3.2
-3.4
-3.6
1960
1970
1980
1990
2000
data
LC
CBD
BLC
Age 90
-1.2
-1.4
-1.6
-1.8
-2
-2.2
-2.4
-2.6
1960
1970
1980
1990
2000
data
LC
CBD
BLC

Males

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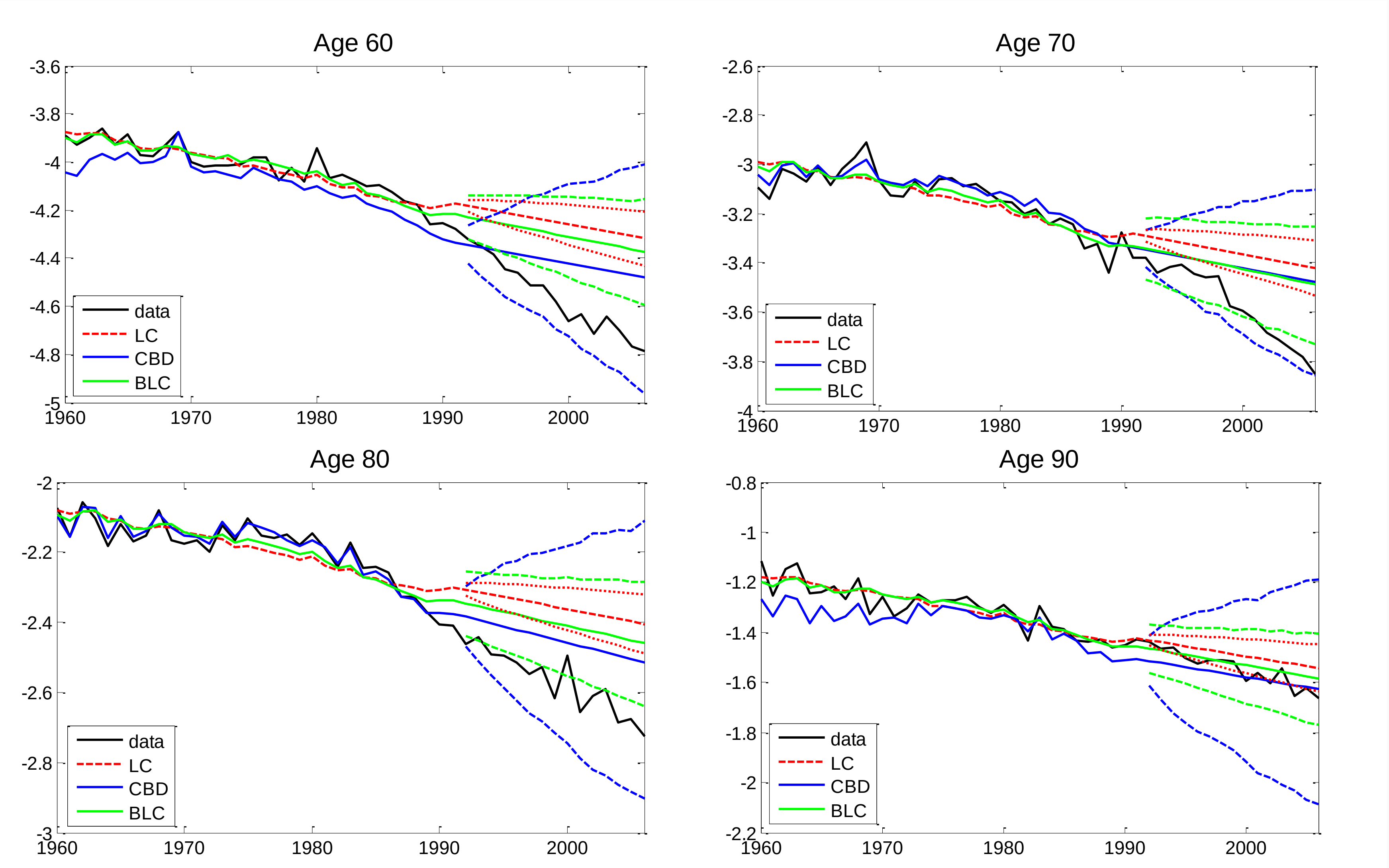
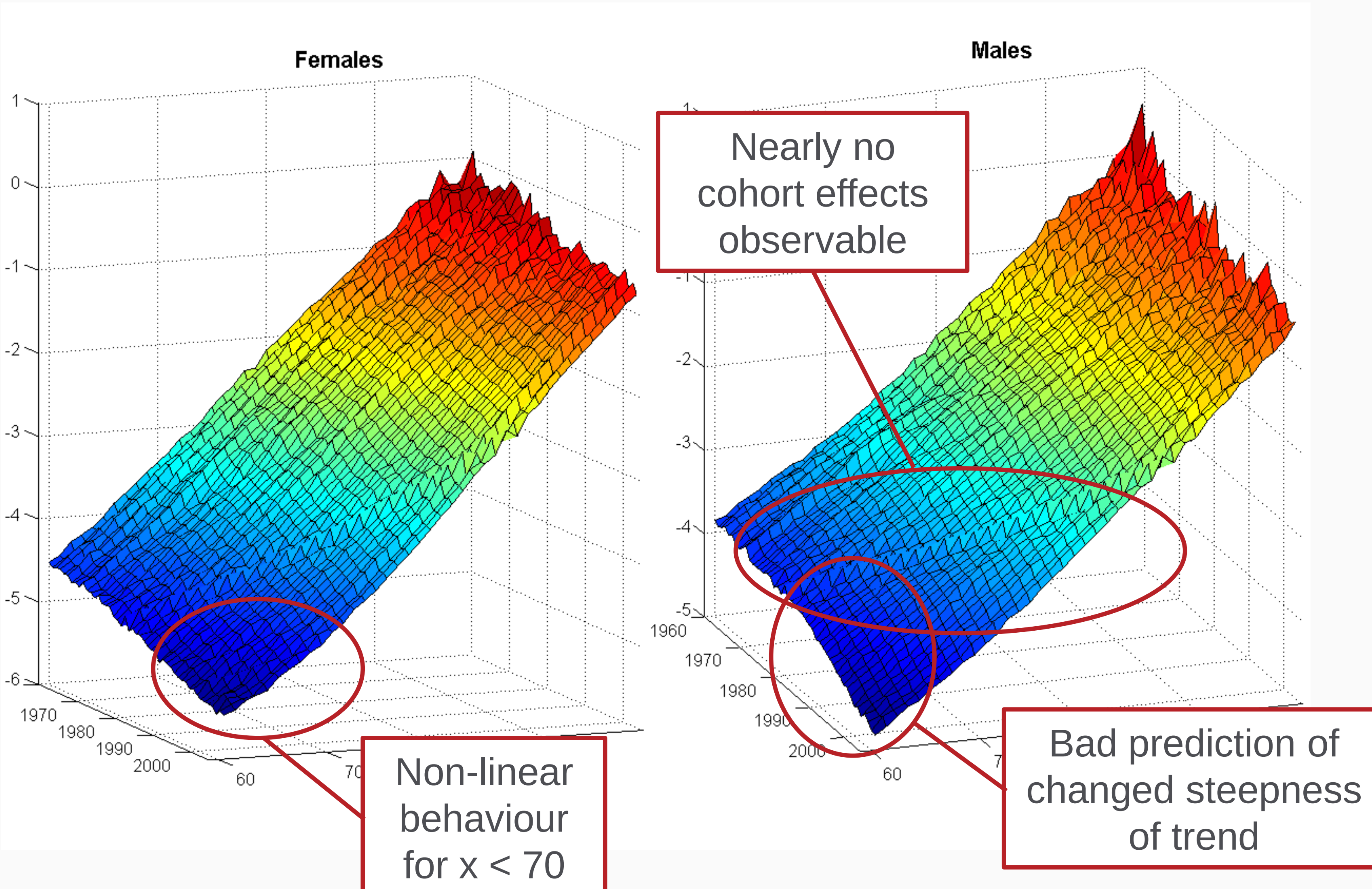


Logit of mortality rates



Discussion

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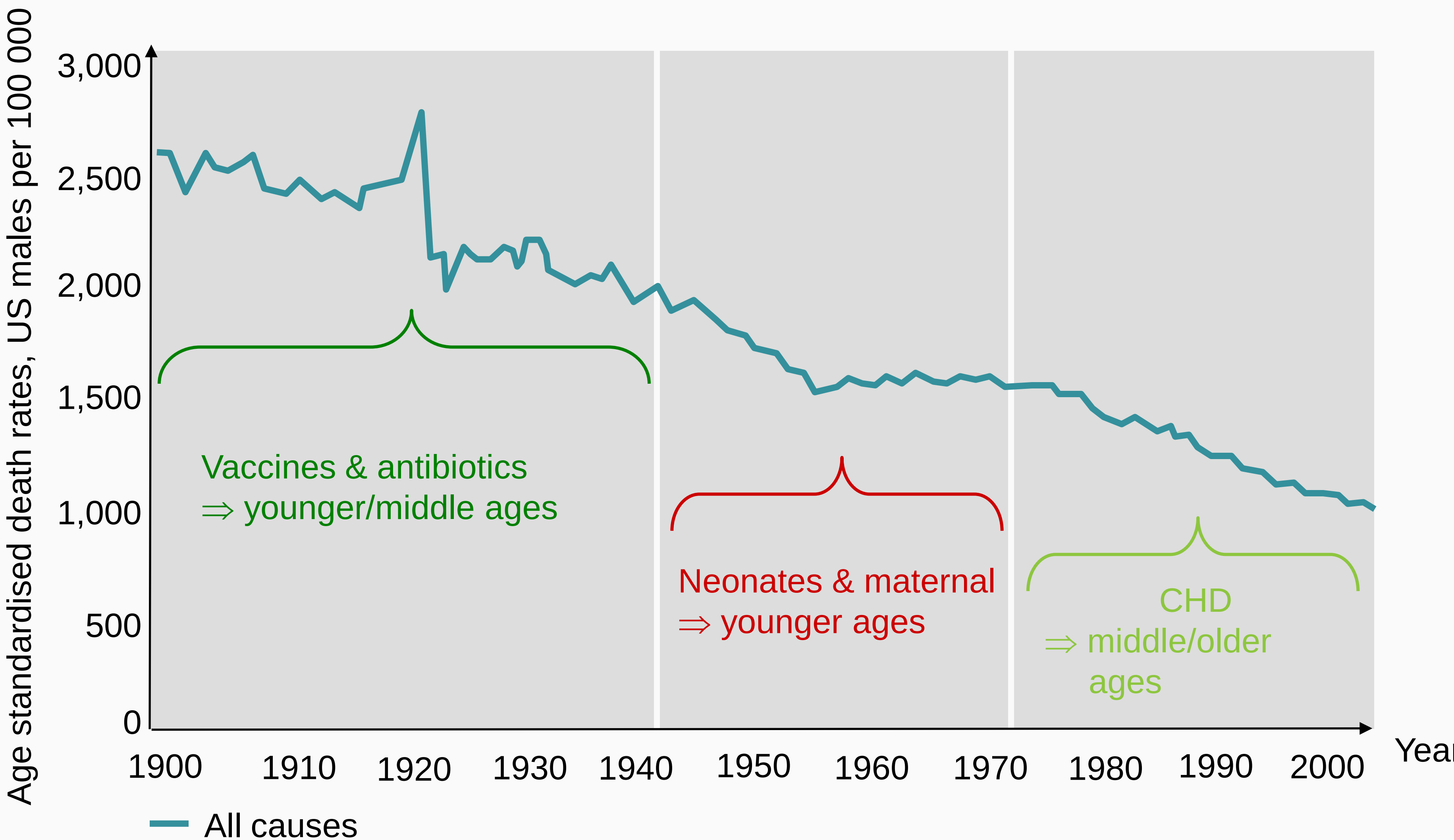
- LC and BLC models capture age effects more accurately
  - The data shows nearly no cohort effects
  - Linearity of logit-transform only valid for ages ≥ 70
- Trend steepness changed around 1985
  - prediction intervals of LC model too small for most ages
  - BLC model leads to more realistic prediction intervals,
    however true mortality rates still outside prediction intervals in some cases
  - CBD model yields the largest prediction intervals:
    true mortality within prediction intervals for all ages,
    but perhaps too conservative?

Development of the Mortality in the 20th century



IMPACT ON ANNUITY PRODUCTS – COMPARISON TO SOLVENCY II

Modeling mortality data from Italy – impact on annuities

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Impact on annuity products

- Fit models to data from 1960 to 2006
- Calculate 99.5% of annuity payments for two settings
  - Immediate annuity payments: portfolio of 1000 65-year olds
  - Deferred annuity payments: portfolio of 1000 30 year olds, 35 years deferment period
- Compare to Solvency II (QIS 5):
  - Longevity Scenario = 75% best estimate mortality for all ages and whole projection period

Mortality projections for 60 years (Females)

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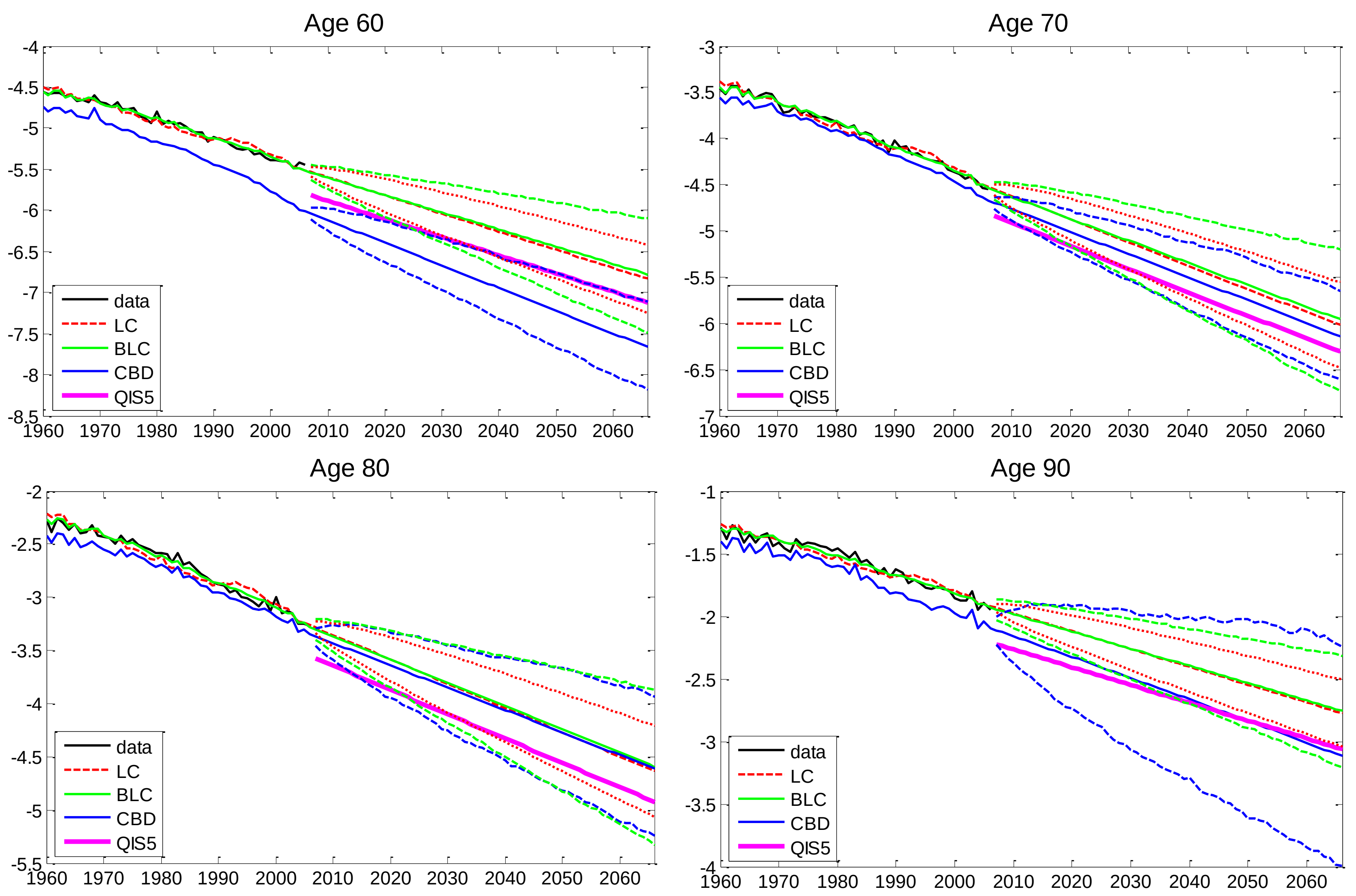

99.5% Percentiles of annuity payments

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Model	Immediate annuity payments starting at age 65		Annuity payments starting at age 65, 35 years deferment	
Females		% b.e.		% b.e
Best Estimate	23.61		27.63	
QIS 5: 75% BE	25.51	+ 8.0%	29.04	+ 5.1%
LC	25.01	+ 5.9%	29.50	+ 6.8%
BLC	25.54	+ 8.2%	30.51	+ 10.4%
CBD	26.79	+ 13.5%	31.76	+ 14.9%
Males				
Best Estimate	18.85		22.53	
QIS 5: 75% BE	21.09	+ 11.9%	24.56	+ 9.0%
LC	20.00	+ 6.1%	24.51	+ 8.8%
BLC	20.71	+ 9.9%	26.00	+ 15.4%
CBD	21.96	+ 16.5%	27.82	+ 23.5%

Model Comparison

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Desirable model features	Lee Carter Baysian Lee Carter Cairns Blake Dowd	QIS 5
Risk capital increasing with projection horizon – to avoid mis-steering business	✓	not the case
Model can be used for calibrating limits and budgets	✓ stochastic model	deterministic model
Ability to model age and cohort effects	can be extended to allow for those effects	not the case
Deeper understanding of future trend developments	not the case	not the case

INTERNAL MODELS IN A SOLVENCY II SETTING

Risk Models under Solvency II

Pillar 1

Quantitative Modelling

- Easy to calibrate
- Consistency between all risk drivers
- Efficiently calculated

Pillar 2

Enterprise Risk Management

- Consistent steering of business
- Calibration of limits and budgets
- Easily applicable in processes

Pillar 3

Transparency and Reporting

- Well understood in industry
- Accepted as best practice
- Certain consistency with former reports

Internal models

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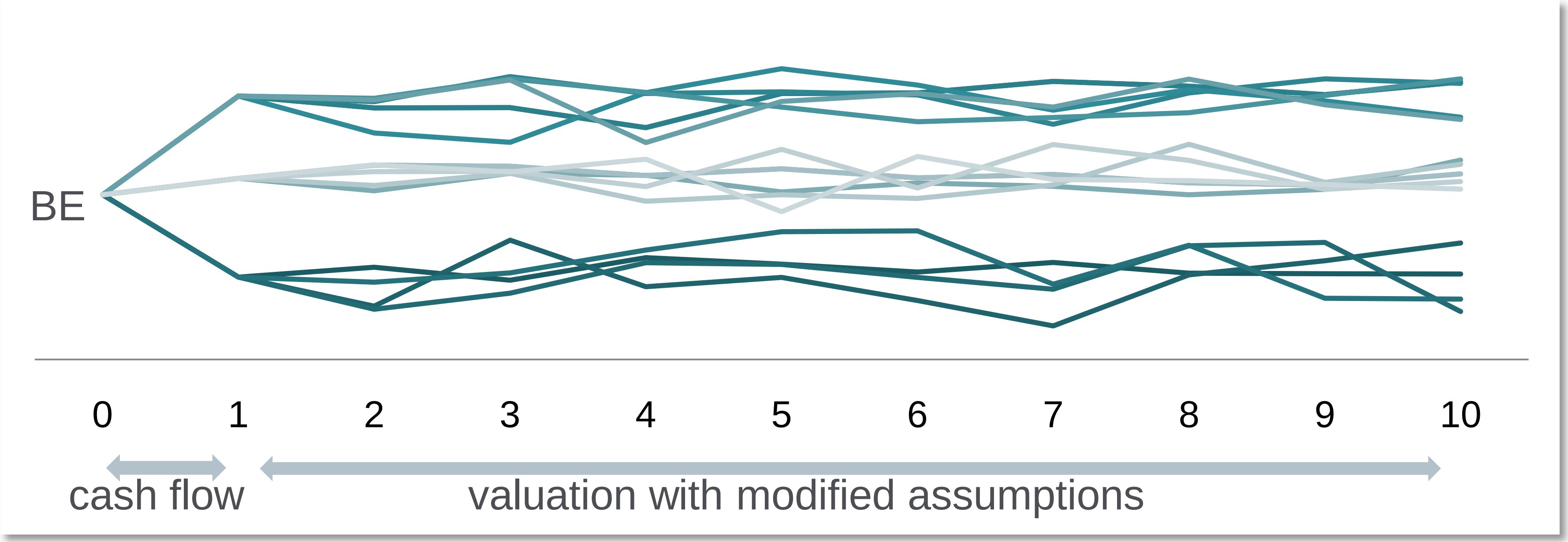
Up to now naive approach: models for run off

Solvency II: SCR = 99.5 % VaR of available capital over 1-year time horizon

SCR = 1-year result +
change of portfolio value due to change of mortality assumptions



Practical implementation

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	1-year result	year 2 and onwards
Complete markets (e.g. financial markets)	real world	risk neutral valuation
Longevity (no hedges available)	real world	q_x real world valuation

► **For non-hedgable risks valuation on martingale measure meaningless!**

Conduct real world valuation by nested simulations:

- Longstaff-Schwartz algorithm
  approach provides stable results for expectation, however not that well suited for estimating quantiles
- Improve approximation by importance sampling and other variance reducing methods like stratified sampling for the valuation of year 2 and onwards

THANK YOU VERY MUCH FOR YOUR ATTENTION

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